Ch. - 20

Volume and Surface area of Solids

Formula for volume and surface area of cuboid and cube.

Cuboid:

1. Volume of cuboid: - length × breadth × height

2. Diagonal of a cuboid = $\sqrt{l^2+b^2+h^2}$

3. Total surface Area = $2(lb+bh+hl)$

4. Lateral surface Area = $[2(l+b)\times h]$

Cube

$V = (edge)^3 = a^3$ cubic units.

Diagonal = $\sqrt{3}a$ units

Q. TSA = $6a^2$ sq. units

LSA = $4a^2$ sq. unit

Find the volume, Lateral Surface area, Total Surface area of cuboid.

length = 22 cm., breadth = 12 cm and h = 7.5

Given that

$l = 22\text{cm}$, $b = 12\text{cm}$, $h = 7.5\text{cm}$.

Volume of cuboid $= l \times b \times h$

$= (22 \times 12 \times 7.5)\ cm^3$

$= 264 \times 7.5$

$= 1980\ cm^3$

Lateral Surface area $= 2 \times [(l+b) \times h]$

$= 2 \times [(22+12) \times 7.5]$

$= 2 \times (34 \times 7.5)$

$= 68 \times 7.5$

$= 510\ cm^2$

Total Surface area of cuboid $= 2[lb + bh + hl)]$

$= 2[22 \times 12 + 12 \times 7.5 + 7.5 \times 22]$

$= 2[264 + 90.0 + 165.0]\ cm^2$

$= 1038\ cm^2$ Ans

Find the volume, the total surface area and the lateral Surface area of cuboid which is 8m long, 6m broad and 3.5 m high.

Given that

length of cuboid = 8m

breadth = 6m

height = 3.5m

Volume of cuboid = $l \times b \times h$

$= (8 \times 6 \times 3.5)\ m$

$= 8 \times \cancel{6}^{3} \times \frac{7}{\cancel{2}}$

$= 8 \times 3 \times 7$

$= 21 \times 8$

$= 168\ m^3$

Total surface Area of cuboid $= 2[lb + bh + hl]\ m^2$

$= 2[8 \times 6 + 6 \times 3.5 + 3.5 \times 8]\ m^2$

$= 2[48 + 21 + 28]\ m^2$

$= 2 \times 97$

$= 194\ m^2$

Lateral surface area of cuboid $= 2[(l+b) \times h]$

$= 2[(8+6) \times 3.5]$

$= 2 \times [\frac{\cancel{14}^{7} \times 7}{\cancel{2}}]$

$= 49 \times 2$

$= 98\ m^2$

Q.3 A cardboard box is 1.2 m long, 72 cm wide and 54 cm high. How many bars of soap can be put into it if each bar measures 6 cm × 4.5 cm × 4 cm?

Sol:-

Cardboard box length = 1.2 m = 1.2 × 100 cm
= 120.0 cm

breadth = 72 cm = 72 cm.

height = 54 cm.

volume of cardboard = l × b × h
= 120 × 72 × 54
= 120 × 54 × 72
= 6480 × 72
= 466560 cm^3

Bar measures (l) = 6 cm
(b) = 4.5
(h) = 4

volume of soap = l × b × h
= 6 × 4 × 4.5
= $\frac{12 \cancel{24} \times 9}{\cancel{2}}$
= 12 × 9
= 108 cm^3

required soap = $\frac{\text{volume of cardboard}}{\text{volume of soap}}$

= $\frac{466560}{108}$

required soap = 4320 Ans

Type-II (Q. 3, 4.)

A wall 15 m long, 30 cm wide and 4 m high is made of bricks, each measuring 22 cm × 12.5 cm × 7.5 cm. If $\frac{1}{12}$ of the total volume of the volume $\frac{1}{12}$ of the wall consists of mortar, how many bricks are there in the wall?

Given that

wall length = 15 m = 1500 cm

breadth = 30 cm

height = 4 m = 400 cm

volume of wall = $l \times b \times h$

$= 1500 \times 30 \times 400$

$= 18000000 \text{ cm}^3$

volume of brick = $(22 \times 12.5 \times 7.5)$ cm

$= 275 \times 7.5$

$= 2062.5 \text{ cm}^3$

volume of mortar = $\frac{1}{12} \times 18000000$

$= 1500000 \text{ cm}^3$

vol. of wall = $18000000 - 1500000 = 16500$

No. of bricks in the wall = $\frac{\text{volume of wall}}{\text{volume of brick}}$

$= \frac{16500000}{2062.5}$

$= \frac{16500000 \times 10}{20625} - 8000$

No. of bricks 8000 Ans

Type- III (Q. 5, 6, 7, 8, 9, 10)

Q.5 Find the capacity of a rectangular cistern in litres whose dimensions are 11.2 m x 6m x 5.8 m. Find the area of the iron sheet required to make the cistern.

Sol:- cistern is a cubodical shape

length of cistern = 11.2 m
breadth = 6 m
height = 5.8 m

volume of cistern = capacity = $l \times b \times h$
$= 11.2 \times 6 \times 5.8$
$= 67.2 \times 5.8$
$= 389.76 \text{ m}^3$

$1 \text{m}^3 = 1000$ litres

volume of cistern = 389.76×1000 litres
$= 389760$ litres.

area of iron sheet = Total surface area of cistern

$= 2(lb + bh + hl)$

$= 2(11.2 \times 6 + 6 \times 5.8 + 5.8 \times 11.2)$

$= 2(67.2 + 34.8 + 64.96)$

$= 2 \times 166.96 = 333.92 \text{ cm}^2$

Hence, area of iron sheet is 333.92 cm^2 Ans

A river 2 m deep and 45 m wide flowing at the rate of 3 km/h. Find the quantity of water that runs into the sea per minute.

Given that

river depth = 2 m

wide = 45 m

Area of the cross section of the river = 2x

= 90

Speed of water = 3 km/hr = 3000 m/

Rate of water flow in 1 minute = $\frac{3000}{60}$ m

= 50 m

Quantity of the water flowing into the sea = area of section x

= 90 x 50

= 4500 m^3 Ans

If the length of each edge of a cube is doubled, how many times does its volume become? How many times does its surface area become?

Given that

let be edge of cube = a

volume of cube = a^3

Surface area of cube = $6a^2$

If each edge will be double, then

volume of cube = $(2a)^3$

$= 8a^3$

Total Surface area = $6 \times (2a)^2$

$= 6 \times 4a^2$

$= 24a^2$

If each ~~eat~~ edge is double volume will be 8 times and surface area 4 times.

Type-7 cube based qus.

The surface area of a cube is 117 cm^2. Find its volume.

Given that

Surface area of cube = 1176 cm^2

$$6a^2 = 1176\ cm^2$$

$$a^2 = \frac{1176}{6}$$

$$a^2 = 196$$

$$a = \sqrt{196}$$

$$a = 14\ cm.$$

volume of cube = a^3

$= 14 \times 14 \times 14$

$= 2744\ cm^3$

The volume of a cube is 729 cm^3. find its surface area.

volume of cube = 729 cm^3

$$a^3 = 729$$

$$a = \sqrt[3]{729}$$

$$a = 9\ cm.$$

Surface area of cube = $6a^2 = 6 \times (9)^2$

$= 6 \times 81$

$= 486\ cm^2$

Q.8 The dimensions of a metal block are 2.25 m by 1.5 m by 27 cm. It is melted and recast into cubes. each of side 45 cm. How many cubes are formed?

Sol:- Given that

dimensions of metal block

$l = 2.25\ m$

$b = 1.5\ m$

$h = 27\ cm = 0.27\ m$

volume of metal block $= l \times b \times h$

$= 2.25 \times 1.5 \times 0.27$

Side of cube $= 45\ cm. = 0.45\ m$

volume of cube $= a^3 = 45 \times 45 \times 45$

$$\text{Required cube} = \frac{\text{volume of metal block}}{\text{volume of cube}}$$

$$= \frac{2.25 \times 1.5 \times 0.27}{0.45 \times 0.45 \times 0.45}$$

$$= \frac{225 \times 15 \times 27 \times 100 \times 100 \times 100}{45 \times 45 \times 45 \times 100 \times 10 \times 100}$$

Required cube $= 10$ Ans

A solid cubical block of fine wood costs ₹ 256 at ₹ 500 per m^3. find its volume and the length of each side.

Total cost of wood = ₹ 256

Per meter cost = ₹ 500

volume of wood = 500

volume of wood × Per meter = Total cost

volume of wood × 500 = 256

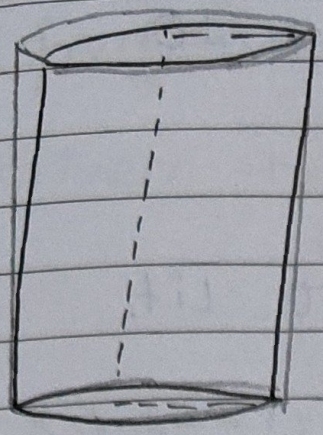
$$\text{volume} = \frac{256}{500} m^3$$

$$= 0.0512 \ m^3$$

$$= 512000 \ cm^3$$

length of each side = $\sqrt{\text{volume}} = \sqrt{512000}$

= 80 cm. Ans



1. Volume of cylinder $= \pi r^2 h$

2. Lateral Surface area $= 2\pi rh$

3. Total Surface area $= 2\pi r(r+h)$

Type-I

A milk tank is in the form of a cylinder whose radius is 1.5 m and height is 10.5 m. find the quantity of milk in litres that can be stored in the tank.

Milk tank

Given that

milk tank radius = 1.5 m.

height = 10.5 m.

Required volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (1.5)^2 \times 10.5$$

$$= \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{15}{\cancel{10}_{2}} \times \frac{\cancel{105}^{3}}{\cancel{10}_{2}} = \frac{99}{2}$$

$$= 49.5$$

$$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{\cancel{15}\ 3}{\cancel{105}}}{\underset{2}{\cancel{10}}}$$

$$= \frac{11 \times 27}{4} \ m^3$$

$$= \frac{11 \times 27 \times 1000}{4} \ \text{Lit.}$$

$297/4 \times 1000 = 742.5 \times 1000$ Lit

$$= 74250 \ \text{litre.}$$

Type-II

Find the height of the cylinder whose volume is $1.54\ m^3$ and diameter of the base is 140 cm?

Given that

volume of cylinder $= 1.54\ m^3$

diameter of base $= 140\ cm = 1.40$ m

$$r = \frac{1.40}{2} = 0.70\ m.$$

volume of cylinder $= 1.54\ m^3$

$$\pi r^2 h = 1.54\ m^3$$

$$\frac{22}{7} \times 0.7 \times 0.7 \times h = 1.54$$

$$h = \frac{\overset{2}{\cancel{154}} \times 7 \times 10 \times 10}{\underset{1}{\cancel{22}} \times 7 \times 7}$$

$$h = 100\ cm = 1\ m.$$

Hence height of cylinder is 1 m. Ans

Q.3 A closed cylindrical tank of diameter 14 m and height 5 m is made from a sheet of metal will be required

Sol:- Given that

diameter of cylindrical tank = 14 m

$r = \frac{14}{2}$

$r = 7$ m.

height = 5 m

Required metal = Surface area of cylinder

$= 2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times 7 (7+5)$

$= 44 \times 12$

$= 528 \ m^2$

Type - III

Q.4 Find the cost of painting 15 cylindrical pillars of a building at ₹ 2.50 per square metre if the diameter and height of each pillar are 48 cm and 7 metres respectively

Sol:- Given that

diameter of pillar = 48 cm = 0.48 m.

height = 7 m

diameter = 0.48 m

radius = $\frac{0.48}{2} = 0.24$ m.

Surface area of pillars = $2\pi r(r+h) = 2 \times \frac{22}{7} \times 0.24 (0.24+7)$

$= 2 \times \frac{22}{7} \times 0.24 (0.24 + 7)$

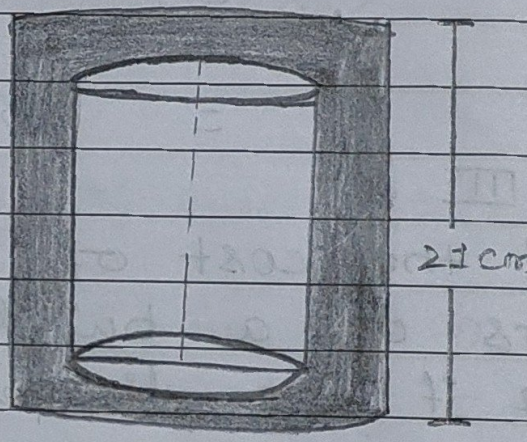
$= 2 \times \frac{22}{7} \times \frac{24}{100} \times \frac{724}{100}$

$= \frac{44 \times 24 \times 724}{7 \times 10000}$

$= ₹\ 396$ Ans.

Type-IV

An iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of pipe is 1 cm and iron weight 8 gm/cm³ find the weight of the pipe.



External radius of the pipe = 4 cm

Thickness of pipe = 1 cm

Internal radius = (4 − 1) cm = 3 cm

External volume = $\frac{22}{7} \times 4 \times 4 \times 21 = 1056$

Internal volume = $\frac{22}{7} \times 3 \times 3 \times 21 = 594\ cm^3$

volume of the metal = external volume − Internal volume

$= (1056 - 594)\ cm^3$

$= 462\ cm^3$

weight of the pipe $= (462 \times 8)\ g$

$= \left(\frac{462 \times 8}{1000}\right) kg$

$= 3.696\ kg.$